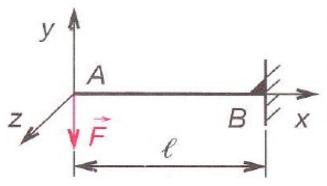
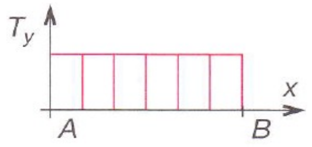
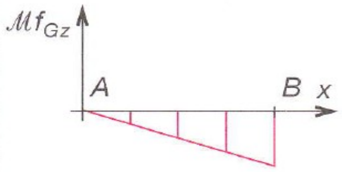
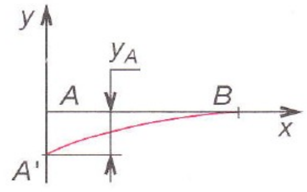
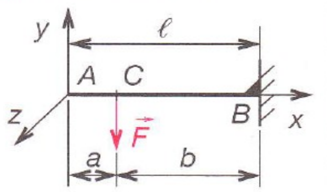
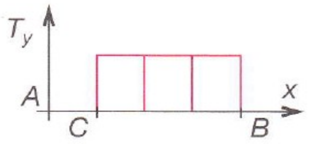
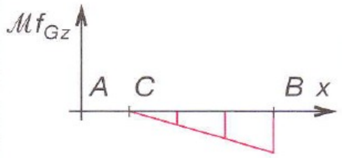
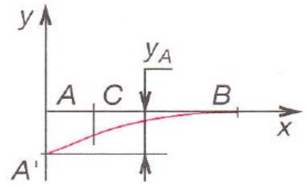
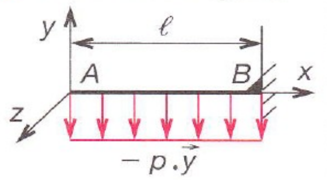
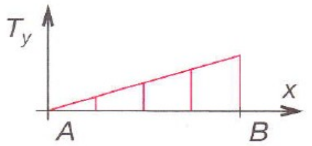
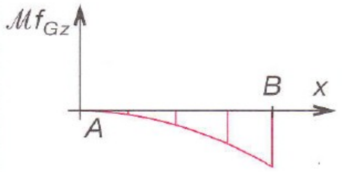
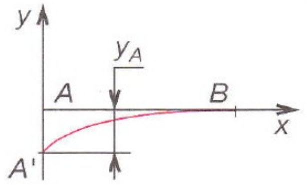
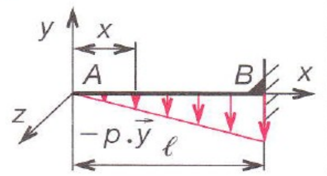
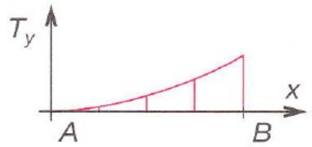
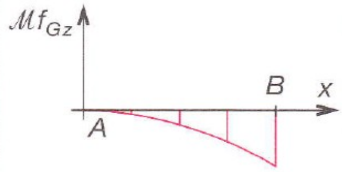
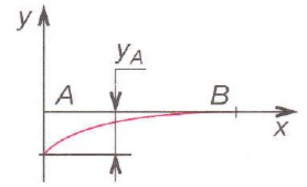
RDM
fiche des contraintes et déformées

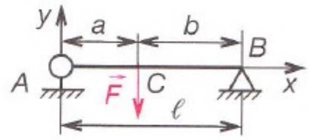
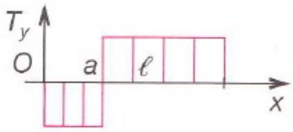
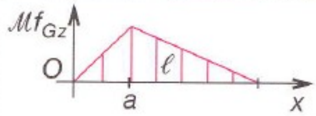
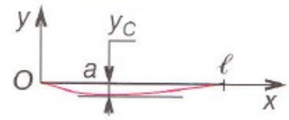
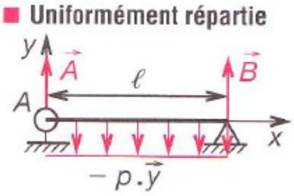
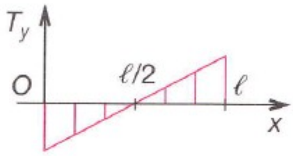
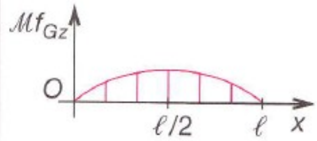
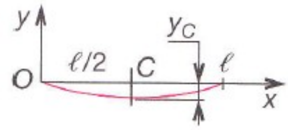
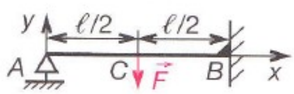
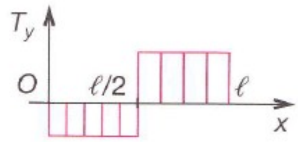
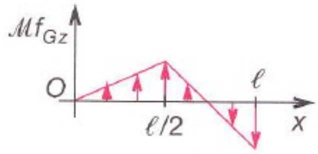
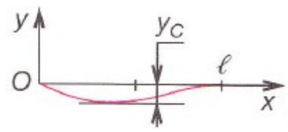
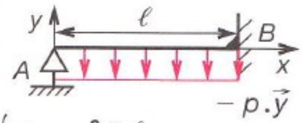
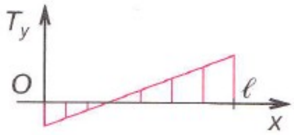
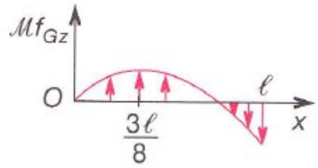
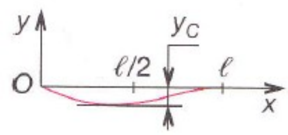
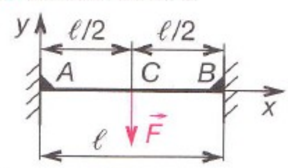
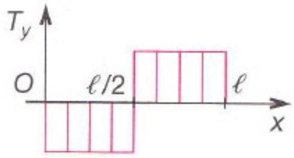
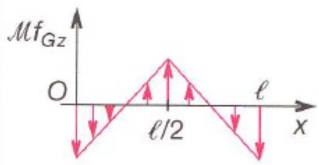
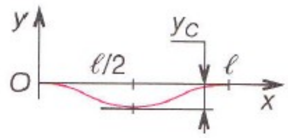
Le 20/01/16
Auteurs : FC, LL

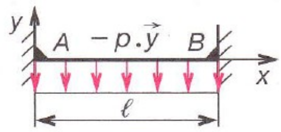
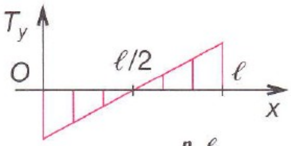
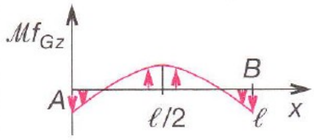
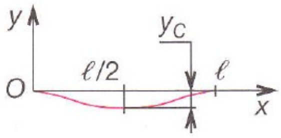
51 ■ 1 POUTRES SUR UN APPUI

Charges – Appuis	Effort tranchant	Moment de flexion	Déformation
■ **Concentrée en *A***  $\vec{B} = -\vec{F} = \lVert\vec{F}\rVert.\vec{y}$ (avec $F < 0$) $\overrightarrow{\mathcal{M}B} = -\lVert\vec{F}\rVert.\ell.\vec{z}$	 Avec $F < 0$ $T_y = +\lVert\vec{F}\rVert$ constant entre *A* et *B*	 Avec $F < 0$ Moment de flexion en *B* : $\mathcal{M}f_{Gz} = -\lVert\vec{F}\rVert.\ell$	 Flèche en *A* : $F < 0$ $y_A = -\dfrac{\lVert\vec{F}\rVert.\ell^3}{3E.I_{Gz}}$
■ **Concentrée en *C***  $\vec{B} = -\vec{F}$ avec $F < 0$ $\vec{B} = \lVert\vec{F}\rVert.\vec{y}$ $\overrightarrow{\mathcal{M}B} = -\lVert\vec{F}\rVert.b.\vec{z}$	 Entre *A* et *C* : $T_y = 0$ Entre *C* et *B* : avec $F < 0$ $T_y = \lVert\vec{F}\rVert$	 Moment de flexion en *B* : avec $F < 0$ $\mathcal{M}f_{Gz} = -\lVert\vec{F}\rVert.b$	 Flèche en *A* : $y_A = -\dfrac{\lVert\vec{F}\rVert(\ell - a)^2(2\ell + a)}{6E.I_{Gz}}$
■ **Uniformément répartie**  *p* : coefficient de charge (N/m) $\vec{B} = p.\ell.\vec{y}$ $\overrightarrow{\mathcal{M}B} = -\dfrac{p.\ell^2}{2}.\vec{z}$	 Effort tranchant max en *B* : $T_{y\,max} = p.\ell$	 Moment de flexion en *B* : $\mathcal{M}f_{Gz} = -\dfrac{p.\ell^2}{2}$	 Flèche en *A* : $y_A = -\dfrac{p.\ell^4}{8E.I_{Gz}}$
■ **Linéairement répartie**  avec $p = k.x$ $\vec{B} = +\dfrac{k.\ell^2}{2}.\vec{y}$ $\overrightarrow{\mathcal{M}B} = -\dfrac{k.\ell^3}{6}.\vec{z}$	 Effort tranchant max en *B* : $T_{y\,max} = \dfrac{k.\ell^2}{2}$	 Moment de flexion en *B* : $\mathcal{M}f_{Gz} = -\dfrac{k.\ell^3}{6}$	 Flèche en *A* : $y_A = -\dfrac{k.\ell^5}{30E.I_{Gz}}$

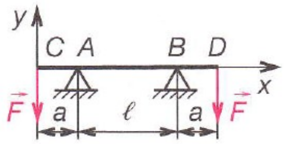
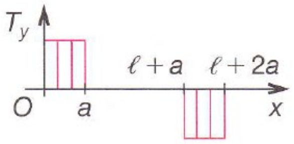
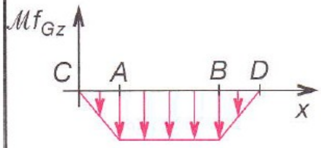
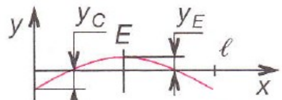
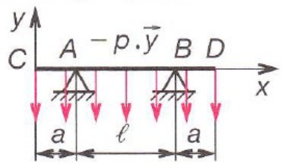
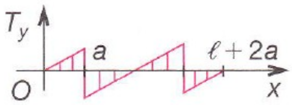
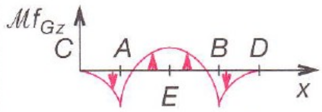
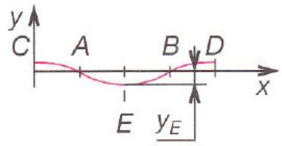
académie Rouen · Sti2D · Lycée J Prévert Pont Audemer	**RDM** **fiche des contraintes et déformées**	Le 20/01/16 Auteurs : FC, LL

51 ■ 2 POUTRES SUR DEUX APPUIS AUX EXTRÉMITÉS

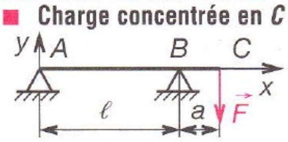
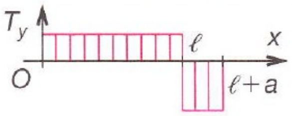
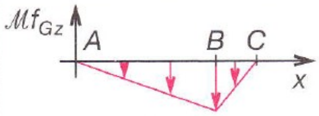
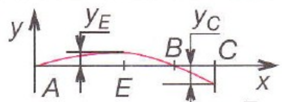
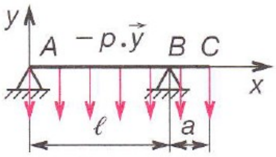
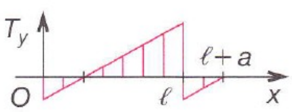
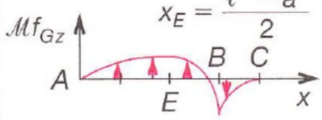
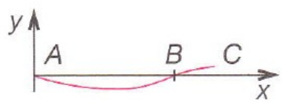
Charges – Appuis	Effort tranchant	Moment de flexion	Déformation
■ **Concentrée en C**  $\vec{A} = \frac{\lVert\vec{F}\rVert.b}{\ell}.\vec{y}$; $\vec{\mathcal{M}A} = \vec{0}$ $\vec{B} = \frac{\lVert\vec{F}\rVert.a}{\ell}.\vec{y}$; $\vec{\mathcal{M}B} = \vec{0}$	 De A à C : $T_y = -\frac{\lVert\vec{F}\rVert}{\ell}.b$ De C à B : $T_y = +\frac{\lVert\vec{F}\rVert}{\ell}.a$	 ■ Pour $x = a$ $\mathcal{M}f_{Gz} = \frac{\lVert\vec{F}\rVert.a.b}{\ell}$ ■ Si $a = \frac{\ell}{2}$ $\mathcal{M}f_{Gz} = \frac{\lVert\vec{F}\rVert.\ell}{4}$	 ■ Pour $x = a$ $y_C = -\frac{\lVert\vec{F}\rVert.a^2.b^2}{3E.I_{Gz}.\ell}$ ■ Si $a = \frac{\ell}{2}$ $y_C = -\frac{\lVert\vec{F}\rVert.\ell^3}{48\,E.I_{Gz}}$
■ **Uniformément répartie**  $\vec{A} = \vec{B} = \frac{p.\ell}{2}.\vec{y}$; $\vec{\mathcal{M}A} = \vec{0}$ $\vec{\mathcal{M}B} = \vec{0}$	 $T_y = +px - \frac{p.\ell}{2}$ En A : $T_y = -\frac{p.\ell}{2}$ En B : $T_y = \frac{p.\ell}{2}$	 $\mathcal{M}f_{Gz}$ est maximal pour $x = \frac{\ell}{2}$ $\mathcal{M}f_{Gz/max} = \frac{p.\ell^2}{8}$	 Flèche en C : $x_C = \frac{\ell}{2}$ $y_C = -\frac{5p.\ell^4}{384\,E.I_{Gz}}$
■ **Concentrée en C**  $\vec{A} = +\frac{5\lVert\vec{F}\rVert}{16}.\vec{y}$ $\vec{B} = +\frac{11\lVert\vec{F}\rVert}{16}.\vec{y}$ $\vec{\mathcal{M}B} = -\frac{3\lVert\vec{F}\rVert.\ell}{16}.\vec{z}$	 De A à C : $T_y = -\frac{5\lVert\vec{F}\rVert}{16}$ De C à B : $T_y = -\frac{11\lVert\vec{F}\rVert}{16}$	 $\mathcal{M}f_{Gz}$ est maximal pour $x = \frac{\ell}{2}$ $\mathcal{M}f_{Gz} = \frac{5\lVert\vec{F}\rVert.\ell}{32}$	 Flèche en C : $y_C = -\frac{7\lVert\vec{F}\rVert.\ell^3}{768\,E.I_{Gz}}$
■ **Uniformément répartie**  $\vec{A} = +\frac{3p.\ell}{8}.\vec{y}$ $\vec{B} = +\frac{5p.\ell}{8}.\vec{y}$ $\vec{\mathcal{M}B} = -\frac{p.\ell^2}{8}.\vec{z}$	 $T_y = px - \frac{3p.\ell}{8}$ En A : $T_y = -\frac{3p.\ell}{8}$ En B : $T_y = \frac{5p.\ell}{8}$	 $\mathcal{M}f_{Gz}$ est maximal pour $x = \frac{3\ell}{8}$ $\mathcal{M}f_{Gz/max} = \frac{9p.\ell^2}{128}$	 Flèche en C : $x = \frac{\ell}{2}$ $y_C = -\frac{p.\ell^4}{192\,E.I_{Gz}}$
■ **Concentrée en C**  $\vec{A} = \vec{B} = \frac{\lVert\vec{F}\rVert}{2}.\vec{y}$ $\vec{\mathcal{M}A} = -\vec{\mathcal{M}B} = +\frac{\lVert\vec{F}\rVert.\ell}{8}.\vec{z}$	 De A à C : $T_y = -\frac{\lVert\vec{F}\rVert}{2}$ De C à B : $T_y = +\frac{\lVert\vec{F}\rVert}{2}$	 $\mathcal{M}f_{Gz}$ est maximal pour $x = \frac{\ell}{2}$ $\mathcal{M}f_{Gz} = \frac{\lVert\vec{F}\rVert.\ell}{8}$	 Flèche en C : $y_C = -\frac{\lVert\vec{F}\rVert.\ell^3}{192\,E.I_{Gz}}$

Charges - Appuis	Effort tranchant	Moment de flexion	Déformation
■ Uniformément répartie			
			
$\vec{A} = \vec{B} = \frac{p.\ell}{2}.\vec{y}$; $\overrightarrow{\mathcal{M}A} = -\overrightarrow{\mathcal{M}B} = \frac{p.\ell^2}{12}.\vec{z}$	$T_y = px - \frac{p.\ell}{2}$; En A : $T_y = -\frac{p.\ell}{2}$; En B : $T_y = \frac{p.\ell}{2}$	$\mathcal{M}f_{Gz}$ est maximal pour $x = \frac{\ell}{2}$; $\mathcal{M}f_{Gz} = \frac{p.\ell^2}{24}$	Flèche en C : $x_C = \frac{\ell}{2}$; $y_C = -\frac{p.\ell^4}{384\,E.I_{Gz}}$

51 ■ 3 POUTRE SUR DEUX APPUIS AVEC PORTE-À-FAUX SYMÉTRIQUE

Charges – Appuis	Effort tranchant	Moment de flexion	Déformation
■ Deux charges concentrées			
			
$\vec{A} = \vec{B} = \Vert\vec{F}\Vert.\vec{y}$; $\overrightarrow{\mathcal{M}A} = \overrightarrow{\mathcal{M}B} = \vec{0}$	Entre C et A : $T_y = \Vert\vec{F}\Vert$; Entre B et D : $T_y = -\Vert\vec{F}\Vert$	$\mathcal{M}f_{Gz}$ entre A et B : $\mathcal{M}f_{Gz} = -\Vert\vec{F}\Vert.a$	■ Flèche en E : $x_E = \frac{\ell+2a}{2}$; $y_E = \frac{\Vert\vec{F}\Vert.a.\ell^2}{8\,E.I_{Gz}}$; ■ En C : $y_C = -\frac{\Vert\vec{F}\Vert.a^2}{6\,E.I_{Gz}}(3\,\ell + 2a)$
■ Charge répartie			
			
$\vec{A} = \vec{B} = \frac{p}{2}(\ell + 2a).\vec{y}$; $\overrightarrow{\mathcal{M}A} = \overrightarrow{\mathcal{M}B} = \vec{0}$	Entre C et A : $T_y = px$; De A à B : $T_y = px - \frac{p(\ell+2a)}{2}$	■ $\mathcal{M}f_{Gz}$ en E : $x = \frac{\ell+2a}{2}$; $\mathcal{M}f_{Gz} = \frac{p}{8}(\ell^2 - 4a^2)$; ■ En A : $\mathcal{M}f_{Gz} = -\frac{p.a^2}{2}$	■ Flèche en E : $x_E = \frac{\ell+2a}{2}$; $y_E = -\frac{p.\ell^4}{16\,E.I_{Gz}}\left(\frac{5}{24} - \frac{a^2}{\ell^2}\right)$

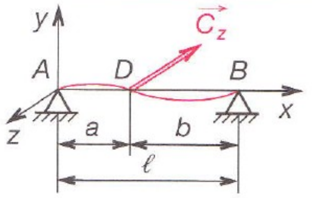
51 ■ 4 POUTRES SUR DEUX APPUIS AVEC PORTE-À-FAUX UNILATÉRAL

Charges – Appuis	Effort tranchant	Moment de flexion	Déformation
■ Charge concentrée en C			
			
$\vec{A} = -\frac{\Vert\vec{F}\Vert.a}{\ell}.\vec{y}$; $\overrightarrow{\mathcal{M}A} = \vec{0}$; $\vec{B} = \frac{\Vert\vec{F}\Vert}{\ell}(\ell + a).\vec{y}$; $\overrightarrow{\mathcal{M}B} = \vec{0}$	Entre A et B : $T_y = \frac{\Vert\vec{F}\Vert.a}{\ell}$; Entre B et C : $T_y = -\Vert\vec{F}\Vert$	$\mathcal{M}f_{Gz}$ en B : $\mathcal{M}f_{Gz} = -\Vert\vec{F}\Vert.a$	■ Flèche en E : $x = \frac{\ell\sqrt{3}}{3}$; $y_E = \frac{\Vert\vec{F}\Vert.a.\ell^2\sqrt{3}}{27\,E.I_{Gz}}$; ■ En C : $y_C = -\frac{\Vert\vec{F}\Vert.a^2(a+\ell)}{3\,E.I_{Gz}}$
■ Charge répartie			
		$x_E = \frac{\ell^2 - a^2}{2}$ 	
$\vec{A} = +\frac{p}{2\ell}(\ell^2 - a^2).\vec{y}$; $\vec{B} = \frac{p}{2\ell}(\ell + a)^2.\vec{y}$	De A à B : $T_y = px - \frac{p}{2\ell}(\ell^2 - a^2)$; De B à C : $T_y = -p(\ell + a) + px$	■ $\mathcal{M}f_{Gz}$ max pour $x = x_E$; $\mathcal{M}f_{Gz} = \frac{p}{8\ell^2}(\ell^2 - a^2)$; ■ En B : $\mathcal{M}f_{Gz} = -\frac{p.a^2}{2}$	

51 ■ 5 POUTRES SUPPORTANT UN COUPLE

Cas 1

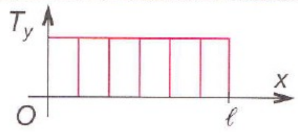
Charges – Déformées



$C_z = C\,;\ C < 0$

$\begin{cases} \vec{A} = -\dfrac{C}{\ell}.\vec{y}\,;\ \vec{B} = +\dfrac{C}{\ell}.\vec{y} \end{cases}$

Effort tranchant

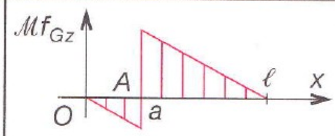


$C < 0$

$0 < x < a \qquad T_y = -\dfrac{C}{\ell}$

$a < x < \ell \qquad T_y = -\dfrac{C}{\ell}$

Moment de flexion



$C < 0\,;\ a \neq 0$

$0 < x < a \qquad \mathcal{M}f_{Gz} = +\dfrac{Cx}{\ell}$

$a < x < \ell \quad \mathcal{M}f_{Gz} = -\dfrac{C(\ell - x)}{\ell}$

Déformation

Flèche en D :

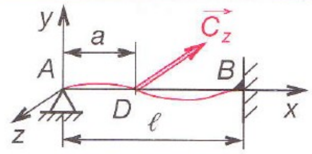
$y_D = \dfrac{1}{E.I_{Gz}} \cdot \dfrac{C.a.b(b-a)}{3\,\ell}$

$\varphi_A = -\dfrac{C}{6\,E.I_{Gz}.\ell}.(\ell^2 - 3b^2)$

$\varphi_B = -\dfrac{C}{6\,E.I_{Gz}.\ell}.(\ell^2 - 3a^2)$

Cas 2

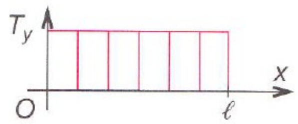
Charges – Déformées



$C < 0$

$\begin{cases} \vec{A} = -\vec{B} = \dfrac{3C}{2\ell^3}(\ell^2 - a^2).\vec{y} \\ \overrightarrow{\mathcal{M}B} = \left|\dfrac{C}{2\ell^2}(\ell^2 - 3a^2)\right|.\vec{z} \end{cases}$

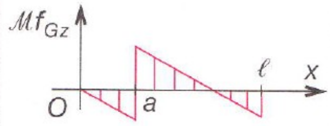
Effort tranchant



$0 < x < a \qquad T_y = -A$

$a < x < \ell \qquad T_y = -A$

Moment de flexion



$C < 0\,;\ a \neq 0$

$\mathcal{M}f_{Gz} = +\dfrac{3C}{2\ell^3}(\ell^2 - a^2).x$

$\mathcal{M}f_{Gz} = -\dfrac{C}{2\ell^3}[2\ell^3 - 3(\ell^2 - a^2).x]$

Déformation

$0 < x < a$

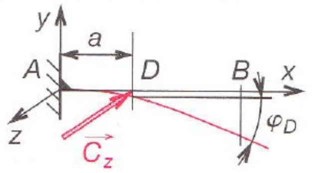
$y = \dfrac{C(\ell - a)x}{4\,E.I_{Gz}.\ell^3} \ldots$

$\ldots[\ell^2(3a - \ell) - (\ell + a)x^2]$

$\varphi_A = -\dfrac{C}{4\,E.I_{Gz}.\ell}(\ell - a)(\ell - 3a)$

Cas 3

Charges – Déformées



$C < 0$

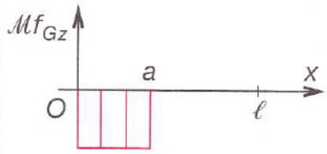
$\begin{cases} \vec{R_A} = \vec{0} \\ \overrightarrow{\mathcal{M}A} = -C.\vec{z} \end{cases}$

Effort tranchant

$x < a \qquad T_y = 0$

$x > a \qquad T_y = 0$

Moment de flexion



$C < 0$

$x < a\,;\quad \mathcal{M}f_{Gz} = +C$

$x > a\,;\quad \mathcal{M}f_{Gz} = 0$

Déformation

$f_D = \dfrac{C\,a^2}{2\,E.I_{Gz}}$

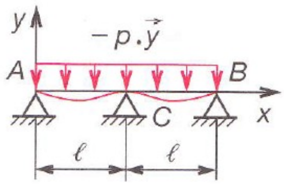
$f_B = \dfrac{C\,a}{E.I_{Gz}}\left(\ell - \dfrac{a}{2}\right)$

$\varphi_D = \dfrac{C\,a}{E.I_{Gz}} = \varphi_B$

51 ■ 6 POUTRES SUR TROIS APPUIS DE NIVEAU

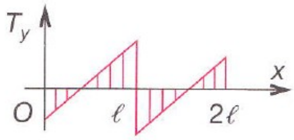
Cas 1

Charges – Déformées



$\begin{cases} \vec{A} = \vec{B} = 0{,}375\,p.\ell.\vec{y} \\ \vec{C} = 1{,}250\,p.\ell.\vec{y} \\ \overrightarrow{\mathcal{M}A} = \overrightarrow{\mathcal{M}B} = \overrightarrow{\mathcal{M}C} = \vec{0} \end{cases}$

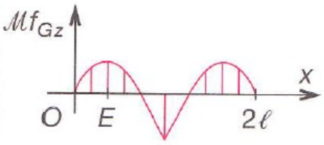
Effort tranchant



$0 < x < \ell \quad T_y = px - 0{,}375\,p.\ell$

$\ell < x < 2\ell \quad T_y = px - 1{,}625\,p.\ell$

Moment de flexion



$0 < x < \ell\,;\quad \mathcal{M}f_{Gz} = 0{,}07\,p.\ell^2$

$\mathcal{M}f_{Gz} = -0{,}125\,p.\ell^2$

$\ell < x < 2\,;\quad \mathcal{M}f_{Gz} = 0{,}07\,p.\ell$

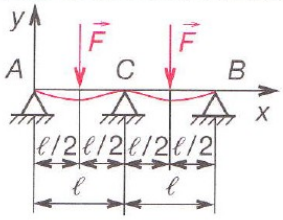
Déformation

Flèche pour $x_E = 0{,}42\,\ell$

$f_E = -0{,}043\,\dfrac{p.\ell^4}{E.I_{Gz}}$

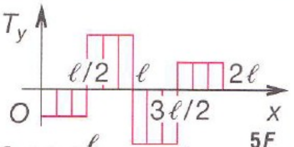
Cas 2

Charges – Déformées



$\begin{cases} \vec{A} = \vec{B} = \dfrac{5F}{16}.\vec{y} \\ \vec{C} = \dfrac{11F}{8}.\vec{y} \end{cases}$

Effort tranchant



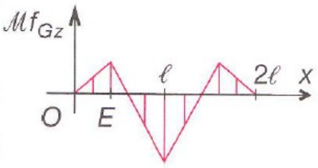
$0 < x < \dfrac{\ell}{2} \qquad T_y = -\dfrac{5F}{16}$

$\dfrac{\ell}{2} < x < \ell \qquad T_y = \dfrac{11F}{16}$

$\ell < x < \dfrac{3\,\ell}{2} \qquad T_y = -\dfrac{11F}{16}$

$\dfrac{3\,\ell}{2} < x < 2\ell \qquad T_y = -\dfrac{5F}{16}$

Moment de flexion



$x = \dfrac{\ell}{2}\,;\quad \mathcal{M}f_{Gz} = \dfrac{5F.\ell}{32}$

$x = \ell\,;\quad \mathcal{M}f_{Gz} = -\dfrac{3F.\ell}{16}$

Déformation

pour $x_E = \dfrac{\ell\sqrt{5}}{5}$

$f_E = -\dfrac{F.\ell^3}{240\,E.I_{Gz}}$